The Perils of Omitting Omissions when Modeling Evidence Accumulation

Xiamin Leng* (xiamin_leng@brown.edu)

Alexander Fengler* (alexander_fengler@brown.edu)

Amitai Shenhav# (amitai_shenhav@brown.edu)

Michael J. Frank# (michael_frank@brown.edu)

Department of Cognitive, Psychological, and Linguistic Sciences,
Brown University,
190 Thayer St, Providence, RI 02912 USA

*: These authors have contributed equally to this work.
#: These senior authors have contributed equally to this work.

Abstract

Choice deadlines are commonly imposed in decision-making research to incentivize speedy responses and sustained attention. However, computational models of choice and response times routinely overlook this deadline, instead simply omitting trials past the deadline from further analysis. This choice is made under the implicit assumption that parameter estimation is not significantly affected by ignoring these omissions. Using new tools from likelihood-free inference, here we elucidate the degree to which omitting omissions, even in seemingly benign settings, can lead researchers astray. Using a Sequential Sampling Model (SSM) with collapsing boundaries as a test-bed, we explore this phenomenon and show how it can be remedied by modeling omissions expected from the generative process.

Keywords: likelihood-free, DDM, SSM, Bayesian Modeling, omissions

Introduction

Joint modeling of choices and response times is a core methodological staple of cognitive science. According to the dominant modeling paradigm, Sequential Sampling Models (SSMs), choices and response times are jointly generated as a result of stochastic evidence accumulation to a boundary (or decision threshold) (Ratcliff, 1978; Ratcliff, Smith, Brown, & McKoon, 2016). SSMs are widely applied, and many variations have been suggested in the literature (Usher & McClelland, 2001; Reynolds & Rhodes, 2009; Hawkins, Forstmann, Wagenmakers, Ratcliff, & Brown, 2015; Malhotra, Leslie, Ludwig, & Bogacz, 2018). However, for all but the simplest assumptions and task design settings, parameter inference for these models can be difficult or computationally intensive. To circumvent these issues, researchers make and collectively accept certain computational shortcuts, under the assumption that they will not have a fundamental impact on the conclusions which can be drawn from experimental studies.

Here, we focus on one such widely applied methodological choice: treating response omissions as missing data, when

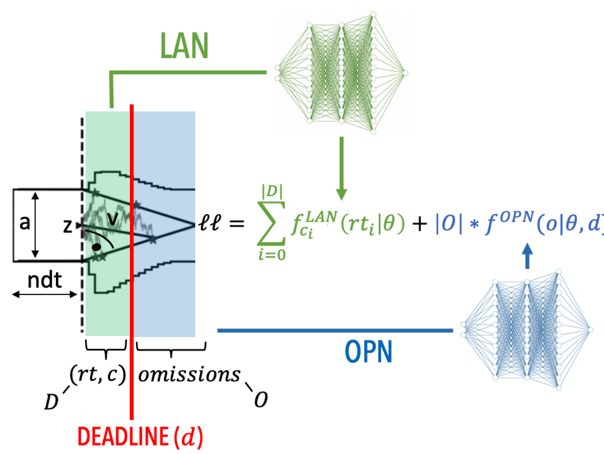

Figure 1: Illustration of the log-likelihood formulation augmenting a LAN with an OPN. The LAN provides a function, $f_c^{LAN}(.)$ for the log-likelihood of observing a given (rt, c) response time and choice pair prior to the deadline. The OPN provides a function, $f^{OPN}(.)$ for the log-likelihood of omissions, given a deadline setting. θ captures the base parameters of the cognitive model.

the rate of omissions is "low". The implicit assumption is that a few omitted trials will not significantly impact parameter estimation. We show that this assumption is invalid and can indeed be pernicious.

Methods

Cognitive Model We focus our analysis here on an SSM with linearly collapsing boundary ("ANGLE" model), often considered appropriate when task designs have deadlines (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006; Frazier & Yu, 2007; Fengler, Govindarajan, Chen, & Frank, 2021). $f^{ANGLE}_{\text{bound}}(t; c, \theta) = a - \left(t * \frac{sin(\theta)}{cos(\theta)}\right)$, which introduces the θ parameter, beyond the *drift rate*, *initial boundary separation*,

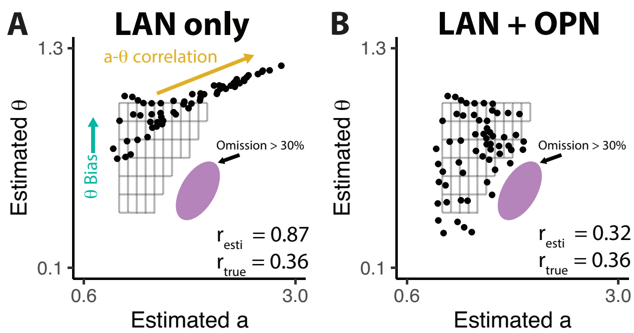

Figure 2: Estimation of the boundary angle θ is contaminated by both estimation biases and magnified a-θ correlation in LAN-only model **(A)** but not in LAN+OPN model **(B)**. Grids represent ground truth parameters. rs are Pearson correlations between a and θ.

starting point bias, and *non-decision time* parameters of the standard Drift Diffusion Model (Ratcliff & McKoon, 2008; Ratcliff et al., 2016).

Likelihoods We use Likelihood Approximation Networks (LANs) to obtain the likelihood of choice,RT tuples (Fengler et al., 2021). To account for the likelihood of omissions, we introduce an *Omission Probability Network* (OPN). We empirically investigate the de-facto vulnerability towards misleading conclusions when *omitting omissions*. Figure 1 showcases the approach to likelihood computation.

Numerical Experiments First, we run a parameter recovery study for the ANGLE model using simulated synthetic datasets, with ground-truth v, z, t fixed ($v = 1.5, z = 0.5, t = 0.3$), and boundary parameters (a, θ) sampled from a 2-D space of realistic values for each of the respective model parameters. We then impose a deadline (1.25s) and exclude parameter sets with too many omissions ($> 30\%$), so that the synthetic datasets generate omission percentages in the range of $0 - 30\%$. We then proceed with two approaches to parameter inference. In the **LAN-only** approach, we simply ignore omissions and evaluate the LAN on observed responses, exemplifying the workflow widely applied in the community. In the **LAN+OPN** approach, we incorporate omissions via the added OPN. The results are shown in Figure 2. We observe a very strong spurious correlation between θ and a when treating omissions as missing data, while accurately eliminating this effect with the inclusion of the OPN.

Second, we run four *synthetic experiments* in which we simulate data across two experimental conditions. In each experiment, the conditions share the same deadline (1.25s) and model parameters (e.g., v, a) except for the boundary collapse parameter θ. In Condition 1, the true boundary collapse is 0.9 rad (larger collapse), while in Condition 2 the collapse is 0.7 rad (smaller collapse). Figure 3 clearly demonstrates the

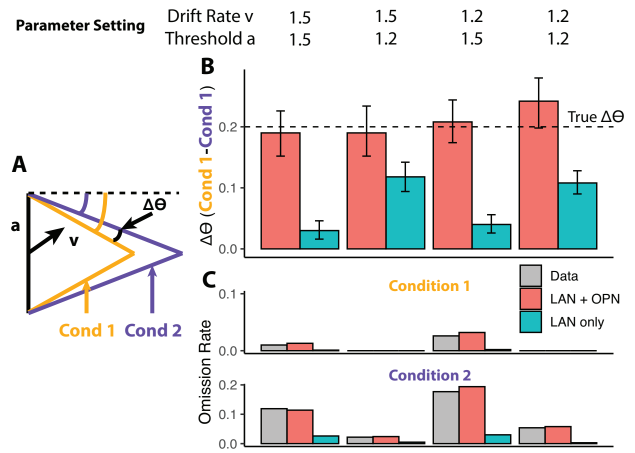

Figure 3: Results of synthetic experiments. **A)** Settings of experiments. Data is simulated from two conditions, with θ values of 0.9 and 0.7 respectively ($\Delta\theta = 0.2$), for a variety of settings of *drift rate* (v) and *threshold* (a) parameters. **B)** Recovery of collapse difference. Ignoring omissions severely underestimates the parameter difference across conditions. **C)** Omission rates in posterior predictive distributions are wrongly calibrated when using the LAN-only approach.

effect of interest ($\Delta\Theta$) is severely misestimated when relying only on a LAN for inference (**B**). Moreover, posterior predictive simulations are grossly miscalibrated as to the expected omission rate (**C**).

Discussion

Enabled by modern computational tools (Fengler et al., 2021; Fengler, Bera, Pedersen, & Frank, 2022), we quantitatively examine the effects of a commonly applied computational shortcut in the context of reaction time and choice modeling: treating omissions as missing data. We come to the following conclusion. As illustrated in Figures 2 even with low omission rates (5% or less), parameter recovery is severely impacted when ignoring omissions at inference. As our synthetic experiment shows (Figure 3), this can result in highly misleading conclusions when comparing parameter values across separate experimental conditions.

We conclude that omissions should not be disregarded, however small their number, when the goal of a study is parameter inference of computational cognitive models, especially (as is commonly the case) the comparison of inferred parameters across groups. On a broader scale, we hope this investigation pointedly shows the value of continued reexamination of methodological choices.

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