Modeling Within-Trial Confidence Trajectories of Reasoning using Quantum Random Walk

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Abstract

In human decision-making, a change-of-mind study analyzes the likelihood of changing the current response choice after conflicting evidence is presented. Several studies have shown that a participant's confidence state associated with the current response may predict change of mind on subsequent responses. Such studies are limited in that they require explicit confidence ratings, within a trial, which can interfere with the cognitive dynamics of response choice. However, computational models can be used to approximate such within-trial dynamics. The present study tested whether the within-trial change in confidence for an initial response is predictive of change of mind on subsequent responses. Within-trial confidence describes the change in confidence state when a stimulus is presented until a response is elicited. A Quantum Random Walk (QRW) model was applied to estimate the within-trial confidence trajectory using a confirmation bias task response accuracy and time. Participants with a higher estimated starting confidence state were found less likely to change their subsequent response choice. Across observed confidence groups (increased, decreased, no change) the estimated QRW confidence states significantly differed for easy and hard trials. Also, the estimated within-trial confidence trajectory for the first response significantly correlated with the observed confidence change between the first and second responses. QRW estimates of individuals' within-trial confidence effectively predicted the individuals' observed confidence states. Thus, QRW as applied to reasoning tasks can effectively model individual differences in within trial confidence trajectories.

Keywords: Quantum Random Walk Model; Bayesian Inference, Change of Mind; Reasoning; Metacognition

Introduction

Change of Mind

In human decision-making, a change-of-mind study investigates scenarios where a participant may change their initial response if conflicting evidence is presented (Stone, Mattingley, & Rangelov, 2022). Several studies (Lundie, 2022; Rollwage et al., 2020) have demonstrated the importance of an individual's confidence state, at the time of eliciting an initial response, in deciding how subsequent conflicting evidence will be processed. For example, Rollwage et al. (2020) showed that participants with a higher confidence state in the initial response are less likely to change their initial response even when presented with conflicting evidence (also known as confirmation bias (Evans, 1989)). However, the confidence state of the decision-maker grows from a starting state when a task is presented to a final state when a response is elicited. Hence, further studies can also be performed to examine the role of this within-trial confidence state trajectory on the likelihood of a change of mind in subsequent responses.

Random Walk models to estimate Within-Trial Confidence State Trajectory

A random walk model over a given state space specifies the likelihood of transitioning from an initial state to a desired state after a given time duration. Quantum random walk models over confidence state space have been studied in the decision-making literature to model change-of-mind (Kvam, Pleskac, Yu, & Busemeyer, 2015; Busemeyer, Kvam, & Pleskac, 2019, 2020). In the Quantum Random Walk (QRW) model, proposed by (Busemeyer, Wang, & Townsend, 2006), the confidence state of a decision-maker is modeled as a superimposition over several states within the state space (described using a wave function). At any given time, the confidence state can be calculated by integrating the likelihood of being in all the states (also known as the collapse of the wave function). QRW models are found to be well applicable to model within-trial confidence evolution of fast decision tasks, especially in case of incongruency in a decision trial (Busemeyer et al., 2019). The current study further examined the applicability of QRW in estimating within-trial confidence trajectories of reasoning tasks.

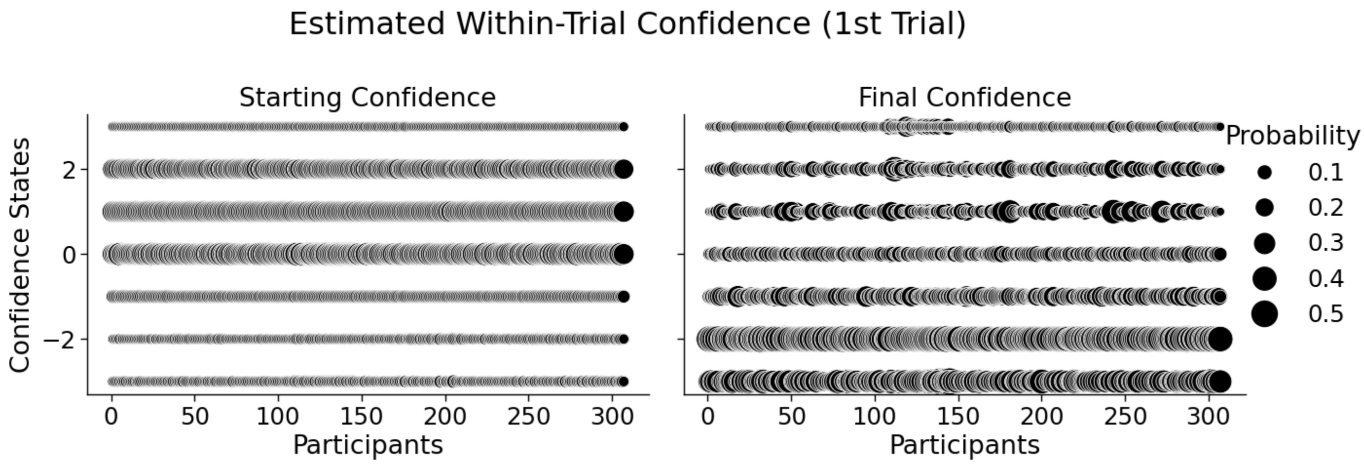

Figure 1: The Staring and Final Confidence shown here describe the estimated superimposed confidence state at the beginning of a trial and at the time of response, respectively. The y-axis describes the 7 confidence states estimated in the current study. A higher confidence state, e.g., +3, describes high confidence in the current response and vice versa.

Current Study

The present study investigated the association of the estimated within-trial confidence state trajectory using QRW with the empirically observed change in confidence ratings reported during an analytical reasoning study conducted by Lundie (2022).

Method

Confirmation Bias Study

Data used for the present study were from a confirmation bias study Lundie (2022). Participants were shown two country flags and asked to select the country with the highest population. Additional evidence was then presented describing the most probable correct response. Participants were instructed to reevaluate their first response while considering the additional evidence and report their second response. Each response was provided along with a confidence rating (e.g., 60L represented 60% confidence left country has a higher population). The task trials varied in degree of difficulty.

Participants A total of 617 participants completed the study conducted by Lundie (2022). The present study removed participants with one or more response times greater than 3 standard deviations from the population's average response time. The remaining 435 participants were further filtered to include participants who did not change their initial response choice for more than 3/4 of the trials. Among the remaining 307 participants, 270 and 274 participants changed their confidence ratings in the second attempt (even though they kept the response choice the same) for easy and hard trials, respectively.

Quantum Random Walk Parameter Estimation

The likelihood function of QRW (with noise), proposed by Busemeyer et al. (2006), was implemented using Python-JAX library (Bradbury et al., 2018). Seven states, [+3,-3], were used to approximate within-trial confidence. A higher state describes a higher confidence in the current response choice. The prior distribution for the confidence state probability distribution was specified using the Dirichlet Distribution. Beta prior was used for the concentration parameter of Dirichlet distribution. Non-centralized priors were used for the rate of confidence accumulation parameter of QRW. 3000 (including 1000 burn-ins) posterior samples were drawn over 4 chains using the No-U-Turn Sampling (NUTS) method (Hoffman & Gelman, 2011) implemented in Python-NumPyro (Phan, Pradhan, & Jankowiak, 2019). The confidence state probability was estimated using the posterior mean and model convergence was evaluated using the $\hat{R}$ metric (Vehtari, Gelman, Simpson, Carpenter, & Bürkner, 2021).

Results

Estimated Within-Trial Starting Confidence State

The present study estimated the initial and final within-trial likelihood of being in a confidence state for a population that did not change their response for more than 3/4 of the trials (Fig 1). QRW estimates suggest that most participants' initial likelihood was higher for mid and higher-confidence states. This suggests that participants with a higher starting confidence state are less likely to change their subsequent responses. Present estimated QRW within-trial findings support observed change of mind results by Rollwage et al. (2020) and provide more granularity in the confidence trajectories without requiring explicit confidence ratings.

Estimated Within-Trial Final Confidence State

Across observed confidence groups (positive, negative, and no change), there was a significant difference between the estimated final confidence state in 1st response. Within easy items, $F(2,201) = 10.63, p < 0.001$, and hard items, $F(2,167) = 4.41, p < 0.05$, participants with a higher estimated positive confidence state increased their observed confidence in the subsequent trial whereas participants in a lower estimated confidence state decreased their observed confidence in the second trial. QRW estimated confidence change in the first trial was positively correlated with observed confidence change between the first and second trials (easy items ($r(305) = 0.174, p < 0.01$) and hard items ($r(305) = 0.154, p < 0.01$)).

Discussion

The present study demonstrated that the Quantum Random Walk model (Busemeyer et al., 2006) can potentially be used to estimate analytical reasoning processes and their confidence trajectories without explicit confidence ratings. We demonstrate with computational granularity that the within-trial confidence trajectory of a participant deliberating their first response impacts the likelihood of changing their subsequent response when further evidence is presented. Future studies can further evaluate the association between individual differences in estimated confidence trajectories and how they differ depending on the degree of confirmatory evidence presented in a given task. Also, further studies are required to investigate the goodness-of-model fit of the Quantum Random Walk model and its generalizability to other analytical reasoning tasks.

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