Choice Stochasticity and Cognitive Imprecision Across Value Tasks

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Abstract

Modern computational models of decision-making acknowledge that choice behavior reflects a stochastic process. Most models of value-based choice assume that value computation is passed through a stochastic function that converts a linear value comparison into choice probability. Stochasticity is typically treated as a noisy random variable and the focus in many of these studies involves evaluating the individual differences in value preferences or value learning. However, less attention has been devoted to understanding the individual differences in choice stochasticity and whether the degree of individual cognitive imprecision is a trait-level characteristic that transfers across different value-based tasks. Here, we evaluate the intra-individual stability of choice stochasticity across two distinct value tasks: a risky lottery task and a delay discounting task, and interrogate the role of cognitive imprecision in accounting for this relationship. We find that regardless of mathematical form, stochasticity correlates across tasks, but the relationship between stochasticity, risk attitudes, and temporal discounting largely depends on the assumption of the choice function. In contrast, the cognitive imprecision models offer precise predictions on the relationship between stochasticity, risk, and discounting across individuals. Thus, cognitive imprecision may serve as a general mechanism that could plausibly account for individual risk attitudes as well as discounting behaviors.

Keywords: choice stochasticity; cognitive imprecision; risky decision-making; delay discounting

Introduction

Stochasticity is a central feature of decision-making. Humans and animals alike behave inconsistently when presented with the same choice option across trials. Models of learning and decision-making attempt to capture choice stochasticity by incorporating a random variable. Despite widespread modeling of stochasticity across many cognitive disciplines, most of the attention is given to parameters representing how agents learn value and manifest preferences (e.g., learning rate, discount factor), reducing stochasticity as a nuisance variable. Previous studies have assessed how these parameters are influenced by individual differences among agents and experimental manipulations [2]. Yet, similar analyses are scarce in the realm of stochasticity [4].

One barrier to analyzing stochasticity is the fact that its nature and source is disputed. Evidence accumulation models posit that noise in decision-making is a stochastic process [8], that could plausibly arise from the intrinsic variability of the choice circuit [3]. Other neurobiologically-plausible computational models suggest that choice variability arises from noise in upstream neuronal representations of value that then feed into winner-take-all choice implementation areas [9][10]. More recently, choice randomness in risky choice has been proposed to arise from noise in the neural representation of magnitudes [1], a process known as cognitive imprecision [6]. An advantage to this theory of stochasticity is that it provides a unitary account of both noisy behavior and risk attitudes.

Thus, it is reasonable to conceive of a common neurocognitive mechanism of imprecision in primary attribute representations that could account for choice randomness in many different types of decisions. Here, we test this hypothesis by analyzing choice stochasticity across two value-based decision-making tasks: one assesing delay discounting, and the other assessing risky lottery preferences. We explore the correlation of randomness across tasks under different models including cognitive imprecision.

Methods

Data were derived from a previous published study [7]. Forty-two medically-healthy, consenting adults completed a risk attitude (RA) and an intertemporal choice (ITC) task. In RA, participants chose between a certain \$5 gain, and a lottery. Lottery amounts and probability varied on a trial-by-trial basis. The ITC task involved a choice between an immediate, smaller monetary amount, and a larger, delayed amount. Similarly, amount and time delay varied on a trial-by-trial basis.

We analyzed behavior from both tasks using two classes of models, namely: **(a)** the linear value difference (VD) model and **(b)** the log-linear cognitive imprecision (CI) model. First, VD uses the standard softmax to fit behavior:

$$Pr(A) = 1 \,/\, (1 + e^{-\gamma(U_{\mathrm{A}} - U_{\mathrm{B}})}) \tag{1}$$

where U_J is the computed values of option $J = \{A,B\}$ and γ measures choice precision (inverse of variability). In contrast, CI assumes the following choice function [5]:

$$Pr(A) = \frac{U_{\mathrm{A}}^{1/\mu}}{U_{\mathrm{A}}^{1/\mu} + U_{\mathrm{B}}^{1/\mu}} \tag{2}$$

where μ measures stochasticity. Rearranging the right-hand side of the equation results in the following log-softmax:

$$Pr(A) = 1/(1 + e^{-\frac{1}{\mu}\log\frac{U_{\mathrm{A}}}{U_{\mathrm{B}}}}) \tag{3}$$

The softmax in VD assumes perfect comparison of value options (utilities) that is eventually injected with noise drawn from a Gumbel distribution. In contrast, CI assumes that noise is scaled with the magnitude of the values. Larger values are noisier than smaller values. CI's softmax assumes that value options are compared logarithmically and that noise is drawn from a log-Gumbel distribution.

Choice behavior in RA was fitted to a power utility model ($U_J = pv^{\alpha}$) while behavior in ITC was fitted using a nonlinear hyperbolic discounting model [7] ($U_J = v^{\alpha}/(1+\kappa d)$). Here, v is the dollar amount, p is the probability of winning, d is the delay to the delivery of v (0 for the immediate option), α measures risk attitudes ($\alpha > 1$, risk-seeking; $\alpha < 1$, risk aversion), and κ is the discount rate. We fitted these models using maximum likelihood estimation in Python and hierarchical Bayesian modelling in R. Correlational analyses were performed in Python as well as in R.

Results

We found that stochasticity was correlated across RA and ITC tasks using both VD (Model 1) ($r(40) = 0.65$, $p < 0.001$) and CI (Model 2) ($r(40) = 0.31$, $p < 0.05$) models (**Fig. 1**). Risk attitudes were correlated with stochasticity using Model 1 ($r(40) = -0.88$, $p < 0.001$), but not Model 2 ($r(40) = 0.22$, $p = 0.15$). Conversely, the discount rate was associated with stochasticity using Model 2 ($r(40) = 0.44$, $p < 0.005$), not Model 1 ($r(40) = -0.005$, $p = 0.97$).

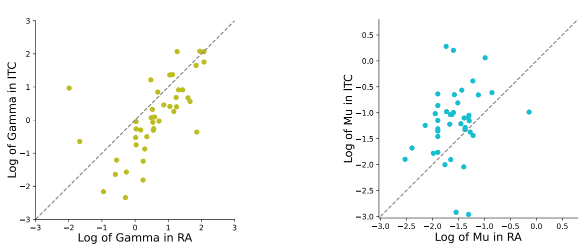

Figure 1: *left*: Stochasticity across economic choice tasks using the VD model. *right*: Stochasticity across economic choice tasks using the CI model.

Here, we test CI's predictions on the relationship between stochasticity and risk/discounting attitudes. First, CI predicts that choice precision is positively correlated with risk-neutrality. Here, we replicate this prediction across the three levels of probability present in the task ($r(40) = 0.91, p < 0.001$ for 25%; $r(40) = 0.74, p < 0.001$ for 50%; and $r(40) = 0.54, p < 0.001$ for 75%). Notably, individuals vary in their risk-seeking behavior: risk-seeking is higher when probability is low relative to high probability. This is consistent with previous findings of more risk-seeking when gains are lower. To establish a common measure of risk attitudes across the probability levels, we take the deviance between risk attitudes from risk-neutral probability. CI predicts a negative relationship between precision and the deviation from risk-neutrality: the smaller the deviation, the closer the individual is to risk-neutrality, and the larger their choice precision. Our results confirm this prediction ($r(40) = -0.81, p < 0.001$) (**Fig. 2, left**).

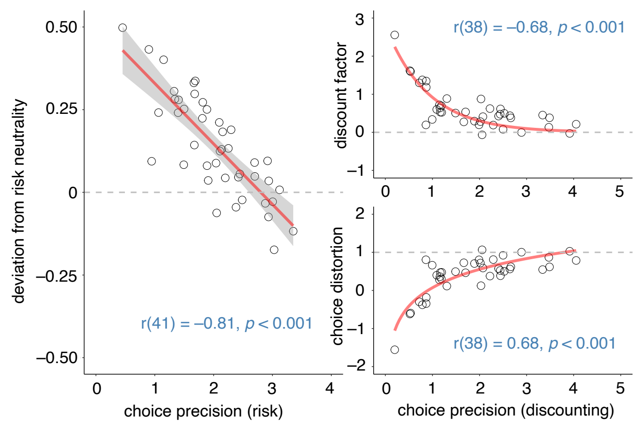

Figure 2: *left*: Correlation between RA choice precision and deviation from risk neutrality. *right, top*: Correlation between ITC precision and discount factor. *right, bottom*: Correlation between ITC precision and comparison distortion between delayed and immediate amount.

Second, we confirm the following predictions in ITC: a negative correlation between choice precision (μ) and the discount rate (κ) ($r(38) = -0.68, p < 0.001$) (**Fig. 2, right top**) and a correlation between delayed-versus-immediate amounts comparison (α) and precision ($r(38) = 0.68, p < 0.001$) (**Fig. 2, right bottom**). Crucially, both κ and α are related. This is consistent with previous findings that show the latent effects of risk attitudes on discounting [7].

Conclusions

Overall, we find evidence that choice stochasticity is conserved across different economic choice tasks. We find that risk attitudes and discount rates are related to their respective stochasticity. Following the CI predictions, η captures the deviations from risk-neutrality. This explains the negative correlation between stochasticity and risk. Finally, we find that the discount factor and the choice distortion between delay and immediate awards are related latent parameters that account for the relationship between discounting, choice distortion, and stochasticity.

Acknowledgments

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