Model Interval Timing with Laplace Neural Manifolds of Past and Future

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Abstract

Recent data suggest that the brain maintains Laplace transformed neural timelines of the past and the planned future. We apply a cognitive model that uses this representation to explain canonical behavior patterns in interval timing tasks. The model comprises three components: 1. a population of exponentially decaying neurons that encode the Laplace transform of past events at various rates across neurons; 2. a weight matrix that stores Hebbian associations of past events with the present; 3. a population of exponentially ramping neurons at various rates that encode the Laplace transform of the expected future given the present. This model allows an agent to continuously update memory and predicted future in relation to the present moment as events unfold. Unlike typical recurrent neural networks (RNNs) for timing tasks, each component in our model maps to concrete cognitive functions, which enables the agent to adjust and manipulate a logarithmically compressed timeline to meet various task demands.

Keywords: interval timing; time perception; neural manifold; cognitive model

Introduction

The ability to track time has been studied extensively in human and other animals, revealing remarkably consistent behavioral patterns across species (Gibbon & Church, 1984; Lewis & Miall, 2009). Early timing models focused on behavior (Gibbon, 1977); more recent computational neural models have used recurrent neural networks (RNN) models that are difficult to map to specific cognitive functions (Laje & Buonomano, 2013; Beiran, Meirhaeghe, Sohn, Jazayeri, & Ostojic, 2023). Here we present a cognitive model based on neural population data that captures continuous timelines for past (Bright et al., 2020; Tsao et al., 2018) and planned future action (Cao, Bright, & Howard, 2024), while integrating previously established cognitive models for timing and memory. The model is applied to various timing tasks to explain canonical behavior patterns.

Temporal Model

The model is formalized in Howard, Esfahani, Le, and Sederberg (2023) where the authors provide an extensive discussion of each components and potential applications. Below we briefly describe each component in the context of interval timing where an agent needs to learn event X is followed by event Y after time T.

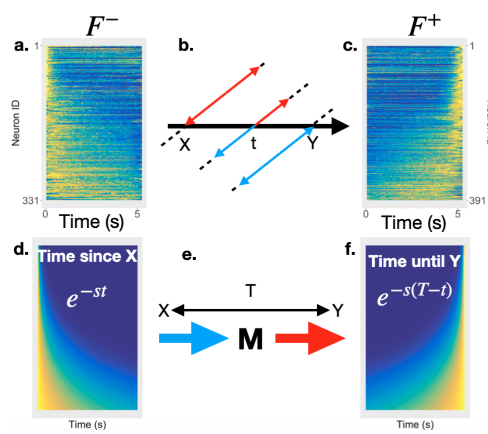

Figure 1: **a-c** Neural representation observed when animals timed their actions for target interval duration (**c**); figure adopted from Cao et al. (2024). **a** Neural population tracks time since the start of interval. Each row represents normalized firing where yellow indicates high firing and deep blue indicates zero firing. **c** neural population tracks time until the end of interval. Plotted the same way as **a**. **d-f** Schematic of the model that learns temporal relationship between two events(**e**).**d** plots simulated neurons carrying a logarithmically compressed Laplace timeline for past following **a**. **f** plots simulated neurons carrying a logarithmically compressed Laplace timeline for future following **c**.

Laplace Transform of Past Timeline F$^-$ When X happens, it is represented as a delta function and activate a population of neurons at that moment where $F^-_{t=t_0}(s) = e^{s0}$. Then each neuron start to decay at each δt exponentially according to:

$$F^-_{t+\delta t}(s) = e^{-s\alpha(\delta t)} F^-_t(s) \tag{1}$$

where the parameter α reflect the ratio of internal time flow over external time flow. Because s takes an continuous spectrum of values sampled from geometric sequence, Eq.1 encodes a logarithmically compressed Laplace timeline of event X in the past.

Hebbian Learning Stored in M The matrix $\mathbf{M}(s)$ stores associations from the past timeline for all events $\mathbf{F}^{-}(s)$ to the future timeline When time T elapsed since event X, event Y occurs. The connection from X in the *past* to Y in the *future* updates: $\rho\mathbf{M}_y^x + (1-\rho)F^{-}_{t=T}(s)$ where ρ controls the forgetting rate of previous trials. Note that $\mathbf{M}(s)$ does not just learn to associate Y with X, but with X in T time ago.

Laplace Transform of Future Timeline $\mathbf{F}^{+}$ We probe $\mathbf{M}(s)$ with the present to generate inputs for the predicted future timeline $F^{+}(s)$. When X occurs again, it activate $\mathbf{F}^{+}(s)$ for Y via $\mathbf{M}(s)$ and start with initial activation copied from $F^{-}_{t=T}(s) = e^{-sT}$ according to eq. 1. Then each neuron start to ramp up according at rate of s:

$$F^{+}_{t+\delta t}(s) = e^{s\alpha\delta t} F^{+}_{t}(s) \tag{2}$$

After applying the initial activation, we get $F^{+}_{t}(s) = e^{s(T-t)}$ where $t = \sum \delta t \alpha$ from the time X occurs again to now. Because $F^{+}_{t}(s)$ for Y peaks when $t = T$, $F^{+}_{t}(s)$ carries out a logarithmically compressed future timeline for predicting Y.

Results

Below we add some simple decision mechanism to the model for three interval timing tasks and report the simulation results.

Interval Reproduction Task

In an interval reproduction task, the agent observes interval T marked by external stimuli and reproduces it by starting and ending an action after T (Fig. 2a). An agent stores the temporal relationship between the start (X) and end (Y) in $\mathbf{M}(s)$ during study. When starting reproducing the interval, the agent probe $\mathbf{M}(s)$ with X', causing $\mathbf{F}^{+}(s)$ for Y' to ramp up. The response time (RT) is defined as the moment when$\mathbf{F}^{+}(s)$ neurons peak, prompting agent to end the interval.

Scalar Property A key feature of interval reproduction is the "Scalar Property", where RT variance increases linearly with interval length(Rakitin et al., 1998). We simulated RT for three intervals (2,4, and 8) over 1000 trials each. For each trial, discrepancy between internal and external time flows, α, is sampled from the same Gaussian distribution (mean=1,SD=0.2). Despite sampled from the same distribution, α multiplies at each time step in $\mathbf{F}^{+}(s)$ and causes the errors to scales with the interval (Fig. 2b,c).

Regression Effect The "Regression Effect", where agents overestimate short duration and underestimate long duration occurs when the study interval varies for each trial (Henke et al., 2021). We simulated RT for 500 trials with intervals randomly chosen between 3 to 7. The model replicate this effect with ρ, a forgetting parameter in $\mathbf{M}(s)$ that incorporate an weighted average of past association between X and Y. This biases the RT towards mean across trials (Fig. 2c).

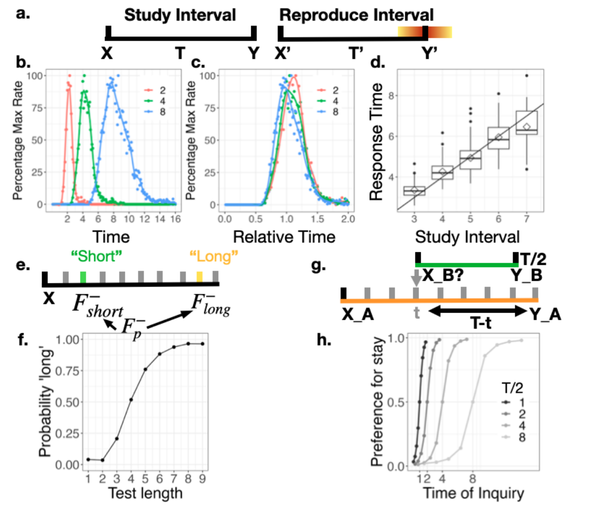

Figure 2: **a** Schematic of interval the reproduction task. An agent timed their action to reproduce studied interval. **b-c** Simulated response time distribution for three intervals plotted at absolute time **b** and rescaled time **c** from the model. **d** Box plot of response time distribution as a function of study interval. Diamond marks the population mean. **e** Schematic of the interval bisection task. **f** Probability or response "long" as a function of test interval length, simulated from the model. **g** Schematic of time left task. **h** Probability of stay as a function of inquiry time for different interval pairs of T and $T/2$.

Indifference Point in Interval Bisection Tasks

In interval bisection task, an agent learned to distinguish between short and long intervals (Fig. 2e) and was tested with intermediate lengths. Researchers found the point of indifference–where agents respond 50/50–is the geometric mean of the two trained intervals (Church & Deluty, 1977).

We model the task as a categorization decision using temporal activities in $\mathbf{F}^{-}(s)$. The activities after short $\mathbf{F}^{-}_{short}(s)$ and long intervals $\mathbf{F}^{-}_{long}(s)$ are stored as the category prototypes An agent then calculate the Euclidean distance of $\mathbf{F}^{-}_{p}(s)$ for a test probe to the two prototypes and assign category accordingly (Smith & Minda, 1998). Because s is sampled according to geometric sequence, the point of equal-distant to the references happens at the geometric mean, rather than arithmetic mean of 2 and 8 (Fig. 2f).

Linear Time in Time-left task

In the Time-left task, an agent learns two intervals: A with reward after T and B with reward after $T/2$. During tests, the agent choose between continuing A or switching to B at different time t during A. Researchers found the indifference point scales with T, which was taken as evidence for linear time perception (Gibbon & Church, 1981).

When B is presented, Our model tracks time left in A and B with $F^{+}_{A}(s) = e^{-s(T-t)}$ and $F^{+}_{B}(s) = e^{-s(T/2-0)}$ respectively. $F^{+}(s)$ ramps up as it gets closer to reward. Therefore we assume that an agent choose stay if $\frac{F_B^+(s)}{F_A^+(s)} < 1$, which is $T/2 - (T-t)$ in the Laplace domain as it is the same as $e^{-s(t-T/2)} < 1$. When $t = T/2$ the agent is indifferent to the choices as $F_A^+(s) = F_B^+$ and this indifference point scales with T despite s is sampled $logarithmically$ (Fig. 2h).

Conclusion

Our results demonstrated that the model effectively captured a wide range of timing behaviors with biologically realistic architectures. It is highly unlikely that generic RNN models for timing could achieve such structured representations. Moreover, the integration of dynamically changing past and future timelines provides a united account for prospective and retrospective timing, addressing gaps in current research.

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