Capacity of the Neuroidal Model for Shared Memory Representations

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Abstract

A central question in neuroscience is how neuronal activity leads to higher level phenomena such as the formation of memories. The Neuroidal model was proposed as a general computational model for brain cognition. This model was later used to suggest how new memories might be created in the mammalian cortex (Valiant, 2005). This was one of many early quantitative theories of memory that used biologically plausible values for parameters such as number of neurons n, number of synapses per neuron d, and inverse synapse strength k. Yet, many fundamental questions remain about the properties of this model. For example, how many memories can be stored given a particular set of parameters? To better understand this question, we offer simple methods for theoretically and empirically evaluating the capacity of the Neuroidal model within specific contexts.

Keywords: neuroids; memory; capacity; machine; learning

Introduction

We choose to closely investigate a basic model of the neural system, known as the *Neuroidal model*, by Leslie Valiant (1994). In this context, neuroids represent biological neurons. These units, their underlying structure, and the model's functional mechanisms are defined in a simple, algorithmic form. We study the capacity of this model to better understand the bounds of computational thought. Also, drawing inspiration from the brain has led to a wealth of innovation in areas such as machine learning (Yang & Wang, 2020), and we intend for our results to inspire such progress.

Background

The Neuroidal model remains a unique perspective on modeling cognition within a machine. It stands separate from the Perceptron-based learning model (McCulloch & Pitts, 1943), which has been widely used today in applications such as deep learning. The model also rejects the notion of cell assemblies, which are a component of the well-established Hebbian learning framework (Hebb, 1949). It is also separate from the Hopfield network (Hopfield, 1982), which has a history of neuroscientific study. We hold interest in the Neuroidal model as a means to better explain the most basic phenomena of the brain, such as unsupervised memorization.

This model has been iterated upon by its author since 1994 and as recently as 2017. Other researchers have also built upon it as a foundation for new neural models with positive results (Papadimitriou, Vempala, Mitropolsky, Collins, & Maass, 2020). However, it seems that only one set of results have been gathered to measure the model's capacity, and they are admittedly limited (Valiant, 2017). Therefore, we have chosen two new avenues for measuring capacity of the model to help mitigate obstacles using previous methods. We have primarily drawn from a much earlier work, Valiant (2005), due to the flexibility of the methods proposed there.

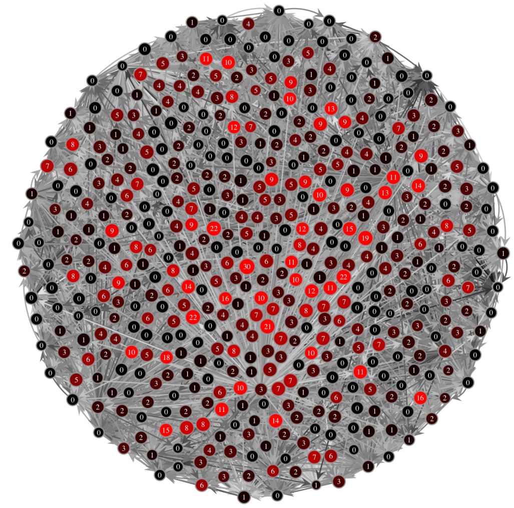

Figure 1: This figure shows an implementation of the Neuroidal model that is at capacity, with $n = 500$, $d = 128$, $k = 16$, $r = 40$. The circles represent neuroids and the gray lines represent synapses. Each number on the graph represents the number of interferences counted as a result of JOIN.

Methods

The Neuroidal model primarily consists of a random graph structure, known as the Erdős-Rényi G_{np} model. In this architecture we have n nodes that are connected by one-way edges, with each possible edge determined to exist at creation by probability p. We assume that each neuron has an expected number of outgoing edges to be d. Each edge has a weighted value, which corresponds to the strength of the synapse of $\frac{1}{k}$. All three values of n, d, and k are determined prior to initialization, and all must maintain quantitative plausibility using current knowledge of neural systems. This allows us to create a brain-like structure with well understood mathematical properties (Valiant, 2005). We offer a toy visualization of such a graph model as Figure 1.

We designate each memory to be a set of neurons, which are expected to be of size r. This value is known as the *replication factor* of the model. This factor was defined in previous work to be subject to several constraints, which were dependent on the specific process used to create memories (Valiant, 2005). In our work, we closely investigate one process, known as JOIN, which creates memories as a result of pre-existing memories like so:

1. Fire all neurons in existing memories A and B.

2. Allow some step(s) of time to pass within the system.

3. Collect a new memory C from neurons that fired.

A simplified example of a structure created from this process is shown as Figure 2. Interference is counted when new memories interfere with previously created memories in the system, a process we will closely describe later. We also make an additional assumption which add to the complexity of this study: Neurons may be *shared* between sets of memories. This assumption introduces several issues, both theoretically and empirically, yet we find this quality of the model to remain plausible of actual neural activity (Komorowski, Manns, & Eichenbaum, 2009). We dedicate this study to circumventing these hurdles by delivering additional mathematical tools and a simulation of the model (Chowdhury, 2023; Perrine, 2023).

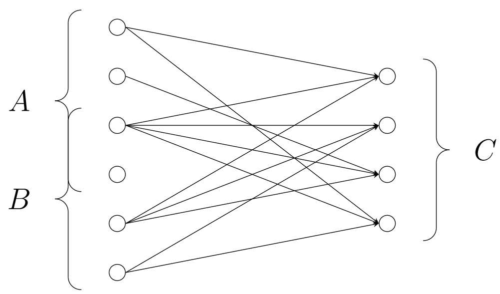

Figure 2: Example of a new memory created by JOIN from shared memories A and B.

Theoretical Results

Memory We define a memory simply as an arbitrary set of nodes within a G_{np} graph with biologically plausible choices of n, d, and k. This concrete coding of discrete *items* remains a plausible method for storing memory in biological systems (Komorowski et al., 2009).

λ-Interference Given two memories U, W, and some number $\lambda \in (0, |W|]$, we say that U λ-interferes with W if

$$|U \cap W| \geq \frac{|W|}{\lambda}.$$

In this case, λ represents a proportion for the amount of allowed neurons between memories. If this number is exceeded, then we consider the two memories to be interfering.

Subset Capacity We will have each memory represented by a subset of the model's vertex set, V. Each memory is expected to be of size r. Locally, we will denote the number of neurons that overlap between two memories as λ. Then we will represent a global interference threshold of T. We also require a fudge factor of δ to account for the variance of r between memories.

Given a set $V = \{v_1, ..., v_n\}$ and parameters $r, T, \lambda, \delta > 0$, the *subset capacity* of V is the *maximum* number of subsets that for any randomly picked memory U,

1. $|U| \in [r - \delta, r + \delta]$,
2. $n >> 2(r + \delta)$,
3. $E[X_U] \leq T$ where X_U is a random variable denoting the number of λ-interferences caused due to picking U.

We prove that the probability that one memory interferes with another memory follows the hypergeometric distribution (Chowdhury, 2023). Therefore, we show that the capacity of the Neuroidal model would be upper bounded by

$$\frac{T}{\sum_{y=\left\lceil \frac{r+\delta}{\lambda} \right\rceil}^{\lfloor r-\delta \rfloor} \frac{\binom{r-\delta}{y}\binom{n-r-\delta}{r-\delta-y}}{\binom{n}{r+\delta}}} + 1.$$

For brevity, we omit our proofs and related theorems here, which are in (Chowdhury, 2023). We clarify that the choices of n, d, and k will affect what r and δ should be when calculating the analytical capacity. Given that the effects of the initialization parameters are difficult to analyze, we also investigated the model in simulation and measured capacity empirically.

Empirical Results

We implemented a simulation of the Neuroidal model. All details are available in a Python Notebook here: https://github.com/patrickrperrine/neural-tabula-rasa-thesis/blob/main/Neuroidal_Model_Simulation_v1_4.ipynb

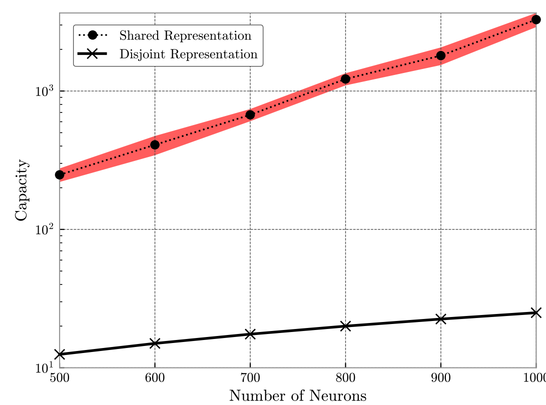

Figure 3: Capacity with different number of neuroids

These results show that the shared memory representation remains worthy of further study, due to the significant increase in capacity when neurons can correspond to different memories. We expect the capacity of the shared representation to continue to grow rapidly when experimenting with larger, more realistic neural models.

Conclusion

We have initiated new methods for better understanding the capacity of the Neuroidal model. Value remains with this model because it works with biological plausible parameters. Although so far we were only able to simulate it for small values of number of neurons, larger simulations could yield interesting insights. These results should inspire further research on the Neuroidal model and related models such as the Assembly Calculus (Papadimitriou et al., 2020).

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